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MULTIVARIABLE ADDITIVE NONPARAMETRIC REGRESSION CURVE ESTIMATIONS TO USE NON-TREND FOURIER SERIES ESTIMATORS

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ABSTRACT

Multivariable additive nonparametric regression model is a nonparametric regression model that involves more than one predictor and has additively separable function on each predictor. There are many functions that can be used on nonparametric regression models, such as the kernel, splines, wavelets, lokal polinomial and fourier series. This research focuses on multivariable additive nonparametric regression models as a result of between non trend fourier series. The estimation method that be used to obtain the estimators is Penalized Least Square. This method requires the estimation of smoothing parameters in the optimation process for obtaining the estimator of multivariable additive nonparametric regression model that has been successfully obtained, which consists of an estimator of fourier series non-trend. The results of this theoretical study shows that the Penalized Least Square method works simultaneously for obtaining the estimators of the smoothing parameters and nonparametric regression models parameters as a result of non-trend fourier series which are additively separable.

Keywords: , Fourier Series, Multivariable nonparametric regression, Non-Trend.

1. Introduction

Parametric regression is the method which is most often used to complete the modeling of the relationship between response and predictor variables when the shape of the functional relationship is known or assumed to follow a specific function. In practice, parametric regression is often not conform to represent the relationship between response and predictor variables because of unknown form of the functional relationship of both variables. In the last decade, it has much to do with studies on nonparametric regression is used to resolve the relationship between response and predictor variables when information is unknown functional relationship. Along with the development of computing and some limitations on the parametric regression model nonparametric regression requires many assumptions that have become more widely applied to solve problems in various fields of applied (Mahler, 1995; Berem and Feng, 2001).

Some research that associated with estimator in nonparametric regression model can be seen in Manzana (2005), Okumura and Naito (2006), Yao (2007), Kayri and Zirhlioglu (2009) for the kernel estimator. Eubank (1999), hang (2006), as well as Budiantara et.al (2010) for the spline estimator. Su and Ullah (2008), He and Huang (2009), as well as Qingguo (2010) for local polynomial estimator. Qu (2003), Rakotomamonjy et.al (2005), and Taylor (2009) for the wavelet estimators. Bilodeau (1992), Faber (2004), Silverberg (2007) and Gaitchouk et.al (2009), Sudiarsa, at.al (2015) for the Fourier series estimator.

Recent developments relating to research on nonparametric regression model shows that the Fourier series is one of the alternative estimator that end - the end is much studied and developed by researchers nonparametric regression. Dc Jong (1997) is a group of early researchers who study

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about the transformation of Fourier series for smoothing function of density, especially in smoothing the spectral estimator. In 1992, Bilodeau examines the Fourier series estimator in nonparametric regression with additive and component predictor Penalized least squares to get the coefficient - coefficient. Fourier series estimator used is the sum of a linear function and a trigonometric polynomial functions.

Among the models of the nonparametric regression, Fourier series is one of the models that have a statistical and a very special and nice visual interpretation (Tripena and Budiantara, 2007). Estimation of Fourier series is capable of handling data that is smooth character and follow the pattern repeated at certain intervals (Bilodeau, 1992). Fourier series that was developed in the nonparametric regression is a Fourier series with the predictor variables have an element of trend up or down trend. On this trend shown by adding one tribe linear equations in Fourier series model. Fourier series estimator developed by the trend was generally very appropriate for the data pattern repeated at certain intervals and follow the trend up or down trend (Bilodeau, 1992, Tripena and Budiantara, 2007). Problem will occur in the use of Fourier series estimator if there is a data pattern repeated at certain intervals, but do not have the rising trend and down trend. It is, therefore, necessary to develop a Fourier series estimator that does not contain the elements of the trend curve estimating nonparametric regression multivariable additive.

1. Formulation Problem
   1.1 How to shape nontrend Fourier series estimator in nonparametric regression curve to estimate multivariable additive.

2. Purposes of Research
   2.1 Getting the form of Fourier series estimator nontrend to estimate multivariable additive nonparametric regression curve.

3. Literature Review
   3.1 Fourier series nontrend Estimator to estimate multivariable nonparametric regression curve.

   Fourier series estimator with multivariable trend, in recent years attracts a lot of attention of several investigators of nonparametric regression. It is generally used when the data pattern is unknown to be investigated and there is a tendency seasonal pattern (Tripena and Budiantara, 2007; Bilodeau, 1992).

   Lets univariable nonparametric regression model $y_1 = g(x_i) + \varepsilon_1$, $i = 1, 2, 3, ..., n$. The from of curve regression $g(x)$ is assumed unknown and contained in a continuous function space. C(0, π). Random error ε_i is assumed to be normally distributed with the independent variant. (σ^2). because $g(x)$ continuous on the interval (0, π), it can be approached by Fourier series with trend function $F(x)$ with:

$$F(x) = bx + \frac{1}{2}a_o + \sum_{k=1}^{K} a_k \cos kx$$

   when b, a_o, ak, $k = 1,2,...,$ K the parameters of model. Based on the Fourier series model. Bilodeau (1992) gives the Fourier series estimator with univariable following trend:

   Teorema 1 (Bilodeau, 1992)

   If the nonparametric function univariable $g(x)$ approached by function $F(x)$, then the Fourier series estimator obtained with the trend of minimize:

$$n^{-1}\sum_{j=1}^{n}\left(y_j + bx - \frac{1}{2}a_o - \sum_{k=1}^{K} a_k \cos kx \right) + w\sum_{k=1}^{K} i^2 a_i^2, w > 0$$

   for each b = $\Re$, $a_o \in \Re$, $a_i \in \Re$, $i = 1,2,...,$ K let by:

$$\hat{F}_w(x) = \hat{b}(w)x + \frac{1}{2}\hat{a}_o(w) + \sum_{k=1}^{K} \hat{a}_k(w)\cos kx$$

   With:

$$\underset{\sim}{\hat{a}}\,(\mathrm{w}) = (\ \left(\hat{b}(w)x, \hat{a}_o(w), \hat{a}_1(w)..., \hat{a}_k(w)\right)'$$

   is obtained from quation :

$$\hat{a}(w) = \left(\hat{b}(w)x, \hat{a}_o(w), \hat{a}_1(w)..., \hat{a}_k(w)\right)'$$

   when D diagonal matrix, let by $D = diag$ (0, 1^4, 2, … K^4) and x a coefisien matrix.

2. Research Method

Obtaining Fourier series estimator nontrend to estimate multivariable nonparametric regression curve.

1. Let a multivariable nonparametric additive regression model:

   $y_i = g_1(x_{1i}) + g_2(x_{2i}) + \ldots + g_p(x_{pi}) + \varepsilon_I$; $i = 1, 2, \ldots, n$

2. Approach regressions curve $g_1(x_1)$, $g_2(x_2)$, $g_p(x_p)$, Fourier series functions nontrend that is:

$$g(x) = \frac{1}{2}a_o + \sum_{k=1}^{K} a_k \cos kx$$

3. Searching for the size of the goodness of fit for optimization PLS:

$$n^{-1}\sum_{i=1}^{n}\left(y_j + \sum_{j=1}^{p} g_j\,(\ _{ji}) \right)^2$$

4. Searching the value of penalty for PLS:

$$\sum_{j=1}^{p}\lambda_j \int_0^{\pi}\frac{2}{\pi} g''_j(x_{ji})^2 dx_{ji}$$

5. Complete the optimization Penalized least Squares (PLS):

$$\underset{g\in C}{Min}\left\{ n^{-1}\sum_{i=1}^{n}\left(y_i - \sum_{j=1}^{p} g_j(x_{ji}) \right)^2 + \sum_{j=1}^{p}\lambda_j \int_0^{\pi}\frac{2}{\pi} g''_j(x_{ji})^2 dx_{ji} \right\}$$

3. Form of Estimator Fourier series Nontrend in Nonparametric Regression curves Estimating Multivariable additive

In this section will be presented nontrend Fourier series estimator in nonparametric regression multivariable additive, which is let by the equation:

$$y_i = \mu(x_{1i}, x_{2i}, ...) + \varepsilon_i$$

$$= \sum_{j=1}^{q} g_j(x_{ji}) + \varepsilon_i, i = 1, 2, ..., n \qquad (6.1)$$

The next regression curve f approached with the function of Fourier series nontrend:

$$g_j(x_{ji}) = \frac{1}{2}a_{oj} - \sum_{k=1}^{K} a_{kj}\cos kx_{ji}, \; j = 1, 2, ..., q \qquad (6.2)$$

Fourier series regression curve estimation nontrend obtained from optimization:

$$\underset{B\in R^{q(k+j)}}{Min}\left\{ n^{-1}\sum_{i=1}^{n}\left(y_i - \sum_{j=1}^{p} g_j(x_{ji}) \right)^2 \right\} \qquad (6.3)$$

With the provision of:

$$J(g_j) = \int_0^{\pi}\frac{2}{\pi}\{g''_j(x_{ji})\}^2 dx_{ji} \le p, p \ge 0 \qquad (6.4)$$

Completion of optimization (6.3) with the condition (6.4) is equivalent to completing the optimization:

$$\underset{B\in R^{q(k+j)}}{Min}\left\{ n^{-1}\sum_{i=1}^{n}\left(y_i - \sum_{j=1}^{p} g_j(x_{ji}) \right)^2 + \sum_{j=1}^{q}\lambda_j \int_0^{\pi}\frac{2}{\pi} g''_j(x_{ji})^2 dx_{ji} \right\}$$

(6.5)

Parameter 2 Parameter is a smoothing parameter that measures the goodness of fit Let by R (g_j) and smoothing (J) (g$_j$)

Fourier series entimator nontread shape obtained from optimization (6.3) let by some of lemma relating to profo of the theorem 1 as next:

Lemma 1

If the given function nontrend fourier series g_j as in equation (6.1), then:

$$J(g_j) = \int_0^{\pi}\frac{2}{\pi}\left(g''_j(x_{ji})\right)^2 dx_{ji} = \sum_{k=1}^{K} k^4 a_{jk}^2, \; j = 1, 2, ..., r.$$

Proof:

because $g_j(x_{ji}) = \frac{1}{2}a_{oj} + \sum_{k=1}^{K} a_{kj}\cos kx_j$, then

$$g_j^n(x_j) = \left(\frac{d}{dx_j}\left(\frac{1}{2}a_{oj} + \sum_{k=1}^{K} a_{kj}\cos kx_j \right) \right)$$

$$= \sum_{k=1}^{K} k^2 a_{kj}\cos kx_j$$

$$\left(g_j^n(x_j)\right)^2 = \left(\sum_{k=1}^{K} k^2 a_{kj}\cos kx_j \right)^2$$

Penalty $J(g_j)$ be a:

$$J(g_j) = \int_0^{\pi}\frac{2}{\pi}\left(\sum_{k=1}^{K} k^2 a_{kj}\cos kx_j \right)^2 dx_j$$

=

$$\frac{2}{\pi}\int_0^{\pi}\left(\sum_{k=1}^{K} k^2 a_{kj}\cos kx_j \right)^2 dx_j + \frac{4}{\pi}\int_0^{\pi}\left(\sum_{k=1}^{K} k^2 a_{kj}\cos kx_j \right)\left(j^2 a_j \cos jx_j\right) dx_j$$

Suppose:

$A = \dfrac{2}{\pi}\displaystyle\int_0^{\pi}\left(\sum_{k=1}^{K} k^2 a_{kj}\cos kx_j \right)^2 dx_j$ and

$$B = \frac{4}{\pi}\int_0^{\pi}\left(\sum_{k=1}^{K} k^2 a_{kj}\cos kx_j \right)\left(j^2 a_j \cos jx_j\right) dx_j$$

Search in advance the value of A and B as next:

$$A = \frac{2}{\pi}\int_0^{\pi}\left(\sum_{k=1}^{K} k^2 a_{kj}\cos kx_j\right)^2 dx_j$$

$$= \frac{2}{\pi}\sum_{k=1}^{K} k^4 a_k^2 \int_0^{\pi}\cos^2 kx_j dx_j$$

$$= \frac{1}{\pi}\sum_{k=1}^{K} k^4 a_k^2\left[x_j + \frac{2}{k}\sin kx_j\right]_0^{\pi}$$

$$= \sum_{k=1}^{K} k^4 a_{jk}^2 \qquad (6.6)$$

The next value of B is let by:

$$B = \frac{2}{\pi}\int_0^{\pi}\sum_{k=1}^{K}\left(k^2 a_{kj}\cos kx_j\right)\left(j^2 a_j \cos jx_j\right)dx_j$$

$$= \frac{4}{\pi}\sum_{k<j}^{K}(kj)^2 a_k a_j \int_0^{\pi}\left(\cos kx_j \cos jx_j\right)dx_j$$

$$= \frac{4}{\pi}\sum_{k<j}^{K}(kj)^2 a_k a_j\left[\frac{1}{k+j}\sin(k+j)x_j + \frac{1}{k-j}\sin(k-j)x_j\right]_0^{\pi}$$

(6.7)

Based on the equation (6.6) and (6.7) were obtained:

$$= J(g_j) = A + B$$

$$= \sum_{k=1}^{K} k^4 a_{jk}^2$$

Lemma 1 can also be presented in the form of a matrix. Matrix form Lemma 1 is let the result of 1.

Result 1:

If $J(g_j)$ let of equation (6.8), then:

$J(g_j) = \underset{\sim}{\beta}'\lambda^* D^* \underset{\sim}{\beta}'$, when,

$$\lambda^* = \begin{pmatrix} \lambda_1 I_{k+2} & 0 & \cdots & 0 \\ 0 & \lambda_2 I_{k+2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_{1r} I_{k+r} \end{pmatrix},$$

$$D^* = \lambda^* = \begin{pmatrix} D & 0 & \cdots & 0 \\ 0 & D & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & D \end{pmatrix} = diag\ (D, D, \ldots, D),$$

$$\beta = \begin{pmatrix} \underset{\sim}{\beta}_1 \\ \underset{\sim}{\beta}_2 \\ \vdots \\ \underset{\sim}{\beta}_r \end{pmatrix} \text{ and } D^* = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 0 & 1^4 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & K^4 \end{pmatrix}$$

Proof:

Based on Lemma is obtained:

$$J(g_j) = \sum_{k=1}^{K} k^4 a_{kj}^2$$

With attention to this equation, Penalty part of the optimization of the equation (6.5) can be represented as:

$$\sum_{j=1}^{q}\lambda_j \int_0^{\pi}\frac{2}{\pi} g''_j (x_{ji})^2 dx_{ji}$$

$$= \lambda_1\int_0^{\pi}\frac{2}{\pi}\left[g_1''(x_1)\right]^2 + \lambda_2\int_0^{\pi}\frac{2}{\pi}\left[g_2''(x_2)\right]^2 + \ldots + \lambda_q\int_0^{\pi}\frac{2}{\pi}\left[g_q''(x_q)\right]^2$$

$$= (\lambda_1, \lambda_2, \ldots., \lambda_4)\begin{bmatrix} \sum_{k=1}^{K} k^4 a_{1k}^2 \\ \sum_{k=1}^{K} k^4 a_{2k}^2 \\ \vdots \\ \sum_{k=1}^{K} k^4 a_{4k}^2 \end{bmatrix}$$

$$= \lambda_1 \lambda_1 \beta_1' D^* \beta_1 + \lambda_2 \beta_2' D^* \beta_2 + \ldots. + \lambda_4 \beta_4' D^* \beta_1$$

$= \beta' \lambda^* D^* \beta$

The addition to need Lemma 1, and result 1, theorem 1 also need Lemma 2 as next:

Lemma 2

If g, the approximated by Fourier series nontrend function (6.2), the goodness of fit $R(g_j)$ can be presented in the form of:

$$R(g) = n^{-1}\left(\underset{\sim}{y} - X(K)\underset{\sim}{\beta}\right)'\left(\underset{\sim}{y} - X(K)\underset{\sim}{\beta}\right),$$

with,

$\underset{\sim}{y} = [y_1, y_2, \ldots, y_n]$,

$$\underset{\sim}{\beta} = \left[\frac{1}{2}a_{01,} a_{11,} \ldots, a_{K1,} \vdots \cdots \vdots \frac{1}{2}a_{01q,} a_{1q}, \ldots, a_{Kzq}\right]$$

$X(K) =$

$$\begin{bmatrix} 1 & \cos x_{11} & \cdots & \cos Kx_{11} & \vdots & \cdots & \vdots & 1 & \cos x_{q1} & \cdots & \cos Kx_{q1} \\ 1 & \cos x_{12} & \cdots & \cos Kx_{12} & \vdots & \cdots & \vdots & 1 & \cos x_{q2} & \cdots & \cos Kx_{q2} \\ \vdots & \vdots & \ddots & \vdots & & & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \cos x_{1n} & \cdots & \cos Kx_{1n} & \vdots & \cdots & \vdots & 1 & \cos x_{qn} & \cdots & \cos Kx_{qn} \end{bmatrix}$$

Proof:

$$R(g) = n^{-1}\sum_{i=1}^{n}\left(y_i - \sum_{j=1}^{q} g_j(x_j)\right)^2$$

$$= n^{-1}\sum_{i=1}^{n}\left(y_i - \sum_{j=1}^{q}\left(\frac{1}{2}a_{oj} + a_{1j}\cos x_{ji} + \ldots + a_{kj}\cos Kx_{ji}\right)\right)^2$$

$$= n^{-1}\sum_{i=1}^{n}\left(y_i - \left(\frac{1}{2}a_{01} + a_{11}\cos x_{11} + \ldots + a_{K1}\cos Kx_{1i}\right) - \ldots + \right.$$
$$\left. - \left(\frac{1}{2}a_{0q} + a_{1q}\cos x_{qi} + \ldots + a_{Kq}\cos Kx_{qi}\right)\right)^2 \quad (6.10)$$

With outline in the form of a matrix equation (6.10) can be written into:

$$R(g) = n^{-1}\left(\underset{\sim}{y} - X(K)\underset{\sim}{\beta}\right)'\left(\underset{\sim}{y} - X(K)\underset{\sim}{\beta}\right), \quad (6.11)$$

The using Lemma 1, result 1 and Lemma 2 forms Deret Fourier estimator nontrend let by Theorem 1 as next:

Teorema 1

If let the regression model presented in the form of equation (6.1) and the Fourier series estimator nontrend obtained from optimization (6.5), then the estimator to function $g_j(x_j)$, j = l, 2, ..., q let by:

$$\hat{g}_j(x_j) = \frac{1}{2}\hat{a}_{oj} + \sum_{k=1}^{K}\hat{a}_{kj}\cos kx_{ji},$$

As a result, estimation for the regression curve $\mu\left(x_{11}, x_{21}, \ldots, x_{qi}\right)$ let by:

$$\hat{\mu}\left(x_{11}, x_{21}, \ldots, x_{qi}\right) = \sum_{j=1}^{q}\hat{g}_j(x_{ji})$$
$$= \sum_{j=1}^{q}\left(\frac{1}{2}a_{oj} + \sum_{k=1}^{K}\hat{a}_{kj}\cos kx_{ji}\right)$$

Proof:

Nontrend Fourier series estimator obtained from optimization:

$$\underset{B\in R^{q(k+j)}}{Min}\left\{n^{-1}\sum_{i=1}^{n}\left(y_i - \sum_{j=1}^{p}g_j(x_{ji})\right)^2 + \sum_{j=1}^{p}\lambda_j\int_0^{\pi}\frac{2}{\pi}g''_j(x_{ji})^2 dx_{ji}\right\}$$

Based on Lemma 2 obtained:

$$R(g) = n^{-1}\sum_{i=1}^{n}\left(yi - \sum_{j=1}^{q}g_{ji}\left(x_{ji}\right)\right)^2 n^{-1}\left(\underset{\sim}{y} - X(K)\underset{\sim}{\beta}\right)'\left(\underset{\sim}{y} - X(K)\underset{\sim}{\beta}\right)$$

The next by Lemma 1 and Result 1 obtained:

$$J(g_j) = \sum_{j=1}^{q}\lambda_j\int_0^{\pi}\frac{2}{\pi}g''_j(x_{ji})^2 dx_{ji} = \underset{\sim}{\beta}'\lambda^* D^*\underset{\sim}{\beta}$$

With combination the goodness of fit $R(g_j)$ and penalty $J(g_j)$, optimization (6.12) can be presented in the form of:

$$\underset{B\in R^{q(k+j)}}{Min} = \underset{B\in R^{q(k+j)}}{Min}\left\{n^{-1}\sum_{i=1}^{n}\left(\underset{\sim}{y} - X(K)\underset{\sim}{\beta}\right)'\left(\underset{\sim}{y} - X(K)\underset{\sim}{\beta}\right) + \underset{\sim}{\beta}'\lambda^* D^*\underset{\sim}{\beta}\right\}$$
$$= \underset{B\in R^{q(k+j)}}{Min}\left\{Q\left(\underset{\sim}{\beta}\right)\right\}$$

Estimator /3 obtained by the partial differentiable Q to /3. The first is described shape Q(/I) as next:

$$Q\left(\underset{\sim}{\beta}\right) =$$
$$n^{-1}\left(\underset{\sim}{y} - X(K)\underset{\sim}{\beta}\right)'\left(\underset{\sim}{y} - X(K)\underset{\sim}{\beta}\right) + \underset{\sim}{\beta}'\lambda^* D^*\underset{\sim}{\beta}$$
$$= n^{-1}\left(\underset{\sim}{y}'\underset{\sim}{y} - \underset{\sim}{y}'X(K)\underset{\sim}{\beta} - \underset{\sim}{\beta}'X'(K)\underset{\sim}{y} + \underset{\sim}{\beta}'X'(K)\underset{\sim}{\beta}\right) + \underset{\sim}{\beta}'\lambda^* D^*\underset{\sim}{\beta}$$
$$= n^{-1}\underset{\sim}{y}'\underset{\sim}{y} - n^{-1}\left(X'(K)\underset{\sim}{\beta}'\underset{\sim}{y}\right)' - n^{-1}\underset{\sim}{\beta}'X'(K)\underset{\sim}{y} + n^{-1}\underset{\sim}{\beta}'X'(K)\underset{\sim}{\beta}'$$
$$+ \underset{\sim}{\beta}'(n^{-1}X'(K)X(K) + \lambda^* D^*)\underset{\sim}{\beta}$$
$$= n^{-1}\underset{\sim}{y}'\underset{\sim}{y} - 2n^{-1}\underset{\sim}{\beta}'X'(K)\underset{\sim}{y} + n^{-1}\underset{\sim}{\beta}'X'(K)\underset{\sim}{\beta} + \underset{\sim}{\beta}'\underset{\sim}{\beta}'(n^{-1}X'(K)X(K) + \lambda^* D^*)\underset{\sim}{\beta}$$

To minimize Q($\underset{\sim}{\beta}$) was obtained by minimizing the partial Q($\underset{\sim}{\beta}$) to $\underset{\sim}{\beta}$ and the result is equated with the zero equation:

$$\frac{\partial Q\left(\underset{\sim}{\beta}\right)}{\partial\underset{\sim}{\beta}} =$$
$$2^{n-1}X'(K)\underset{\sim}{y} + 2^{n-1}\underset{\sim}{\beta}'X'(K)\underset{\sim}{\beta} + 2(^{n-1}X'(K)X(\underset{\sim}{K}) + \lambda^* D^*)\underset{\sim}{\beta}$$
$$= 2(n^{-1'}X''(K)\underset{\sim}{y} + (n^{-1'}X''(K)X(\underset{\sim}{K}) + n^{-1}X'(K)X(\underset{\sim}{K}) + \lambda^* D^*)\underset{\sim}{\beta}$$

Be equating the $\dfrac{\partial Q\left(\underset{\sim}{\beta}\right)}{\partial\underset{\sim}{\beta}} = 0$ equation obtained as next :

$$\left(-n^{-1}X'(K)\underset{\sim}{y} + \left(n^{-1}X'(K)X(\underset{\sim}{K}) + n^{-1}X'(K)X(\underset{\sim}{K}) + \lambda^* D^*\right)\underset{\sim}{\beta}\right)$$
$$\left(-X'(K)\underset{\sim}{y} + \left(X'(K)X(\underset{\sim}{K}) + X'(K)X(\underset{\sim}{K}) + n\lambda^* D^*\right)\underset{\sim}{\beta}\right)$$

With assume elaboration and matrix X (K) nonsingular (matrik with full rank) then obtained estimator $\underset{\sim}{\beta}$ as next:

$$\hat{\underset{\sim}{\beta}}(K) = \left(X'(K)\underset{\sim}{X}(K) + X'(K)\underset{\sim}{X}(K) + n\lambda^* D^* \right)^{-1} X'(K)\underset{\sim}{y}$$

$$= \left[\frac{1}{2}\hat{a}_{01}, \hat{a}_{11}, \ldots \hat{a}_{K1} \vdots \cdots \vdots \frac{1}{2}\hat{a}_{01q}, \hat{a}_{11q}, \ldots \hat{a}_{Kq} \right]$$

Estimator for the regression curve g (x1) let by:

$$\hat{g}_j(x_j) = \frac{1}{2}\hat{a}_{oj} + \sum_{k=1}^{K} \hat{a}_{kj} \cos kx_{ji},$$

A result estimates for the regression curve μ (x_{1i}, x_{2i}, ..., x_{qi}) let by:

$$\mu(x_{1i}, x_{2i}, \ldots, x_{qi}) = \sum_{j=1}^{q} \hat{g}_j(x_j) = \sum_{j=1}^{q} \left(\frac{1}{2}\hat{a}_{oj} + \sum_{k=1}^{K} \hat{a}_{kj} \cos kx_{ji} \right)$$

4. Conclusion

Based on the result and discussion that has been done so can be concluded as next:

Nonparametric regression curve estimator multivariable Fourier series nontrend additives derived from optimization:

$$\underset{B \in R^{q(k+j)}}{Min} \left\{ n^{-1} \sum_{i=1}^{n} \left(y_i - \sum_{j=1}^{q} g_j(x_{ji}) \right)^2 + \sum_{j=1}^{q} \lambda_j \int_0^{\pi} \frac{2}{\pi} g''_j(x_{ji})^2 dx_{ji} \right\}$$

This optimization result in Fourier series estimator nontrend:

$$\mu(x_{1i}, x_{2i}, \ldots, x_{qi}) = \sum_{j=1}^{q} \hat{g}_j(x_j) = \sum_{j=1}^{q} \left(\frac{1}{2}\hat{a}_{oj} + \sum_{k=1}^{K} \hat{a}_{kj} \cos kx_{ji} \right)$$

With $\hat{a}_{oj}, \hat{a}_{kj}$, j = 1, 2, … ,q, k = 1, 2, …, K let by equations:

$$\hat{\underset{\sim}{\beta}}(K) = \left(X'(K)\underset{\sim}{X}(K) + X'(K)\underset{\sim}{X}(K) + n\lambda^* D^* \right)^{-1} X'(K)\underset{\sim}{y}$$

$$= \left[\frac{1}{2}\hat{a}_{01}, \hat{a}_{11}, \ldots \hat{a}_{K1} \vdots \cdots \vdots \frac{1}{2}\hat{a}_{01q}, \hat{a}_{11q}, \ldots \hat{a}_{Kq} \right]$$

5. Reference

Bilodeau, M., 1992, Fourier Smoother and Additive Models, *The Canadian Journal of Statistics*, 3, 25 7-269.

Budiantara, I. N., Lestari, B., Islamiyati, A., 2010, *Estimator Spline Terbobot dalam Regresi Nonparametrik dan Semiparametrik Heteroskedastisitas untuk Data Longitudinal*. DP2M Dikti. Jakarta.

De Jong, P., l977, The Fast Fourier Transform Spectral Estimator, *Journal of Statistical Soe. B 39*, 3, 327-330.

Eubank, R. L., 1999, *Nonparametric Regression and Spline Smoothing*, Second Edition, New York, Marcal Dekker, Inc.

Faber, D., Douglas, C. Yoon, Susan K., Service and Struat C. White., 2004, Fourier and Wavelet Analysis of Dental Radiographs Detect Trabecular Changes in Osteoporosis Bone, 35, 403-411.

Galtchouk. and Pargemenshcikou S., 2009, Adaptive Asymptotically Efficient Estimator in Heterosceastic Nonparametric Regression, *Journal of The Korea Statistical Society*, 38, 3 05-322.

He, H. and Huang, L. S., 2009, Double Smoothing for Bias Reduction in Lokal Linier Regression, *Journal of Statistical Planning and Inference*, 139, 1056-1072.

Kayri, M., and Zirhlogu, G., 2009, Kernel Smoothing and Coosing Bandwith for Nonparametric Regression Methods, *Ozean Journal of Applied Sciences*, 2, 49-54.

Liang, H., 2006, Estimation in Partially Linier Models and Numerical Comparisons, *Computational Statistics and Data Analysis*, 50, 675-687, DOI 10.101617.CSDA.2004.l0.007.

Mahler, 1995, Variational Solution of Penalized Likelihood Problem and Smooth Curva, *Annal of Statistic*, 23, 1496-1517.

Manzan, S. and Zerom, D. 2005, Kernel Estimation of a Partially Linier Additive Model, *Statistic and Probability Letters*, 72, 313-322.

Okumura, H. And Naito, K., 2006, Nonparametric Kernel Regression for Multinomial Data, *Journal of Multivariate Analysis*, 97, 2009-2022.

Qu, L., 2003, On Semiparametric Regression Via Wavelet, Disertation, Purduc University.

Rakotomamonjy, K., Mary, X. and Canu, S., 2005, Nonparametric Regression with Wavelet Kernels, Appl. Statistic Models in Business and Industry., 21, 153-163, l0.1002/asmb.533.

Su, L., and Ullah, A., 2008. Local Polynomial Estimation of Nonparametric Simultaneous Equations Models, *Journal of Econometrics*, 144, 193-218.

Sudiarsa I Wayan., Budiantara, Z.N., Suhartono, Purnami S.W, Combined Estimator Fourier Series and Spline Truncated In Multivariable Nonparametric Regression, *Applied Mathematical Sciences*, Vol. 9, 2015, No.100, 4997- 5010.

Tripena, A. and Budiantara, I N., 2007, *Fourier Estimator in Nonparametric Regression*, *International Conference on Natural Sciences and Applied Natural Sciences*, Ahmad Dahlan University, Yogyakarta.